the rotating airfoils are much higher than that for stationary airfoils in two-dimensional flow. The effect can be traced out to be due to the separation being delayed at larger angle of incidence. The displacement of separation is explained by Dwyer and McCroskey to be due to the appearance of a favorable pressure-gradient, cross-flow derivative, Coriolis forces and centrifugal forces. The Coriolis forces and apparent pressure-gradient are induced by the potential cross-flow. The maximum benefits with regard to separation are due to favorable pressure-gradient, and it will be more when close to the axis of rotation. The detailed explanation is given in Ref. 2.

The slope of the lift curve for two-dimensional flow is larger than the rotating case. Therefore, at smaller angles of attack the stationary blade in two-dimensional flow gives higher lift-coefficient than the rotating one, and is better in performance at those angles. However, at large angles of attack, the rotating airfoil has a better lift-coefficient.

Conclusions

From the results of the investigation the following conclusions can be drawn: 1) The over-all lift coefficient decreases with the increase of blade length. 2) The slope of lift curve for two-dimensional case is larger than the rotating. Lift coefficients at large angles of attack are greater for the rotating case. The rotating blade stalled at higher angle of attack than a stationary blade. 3) The present study confirms the experimental results obtained by Himmelskamp and theoretical results by Dwyer and McCroskey. Hopefully the present study will help pave the way for an extended study of rotating airfoils.

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Airplane Yaw Perturbations due to Vertical and Side Gusts

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An apparently well-accepted fact pertaining to the analysis of aircraft flight in continuous random turbulence is that the main disturbance to lateral tracking is due to side gusts, and further, that the main contribution of the side gust is accounted for by the sideslip it induces. Consequently, the "beta gust" often has been the sole disturbance input considered in recent airplane turbulence performance studies.[1] Actually, a side gust distribution induces a rigid-body response of the airframe not only by producing an effective sideslip at its center of gravity, but also by producing an effective yaw angular velocity through the linear gradient of the side gust along the aircraft longitudinal axis.[2] Both of these effects must be considered simultaneously if the resultant airframe perturbation in a continuous random side gust is to be realistically determined. As will be shown, the yaw-rate effect of the gust produces considerable low-frequency attenuation of the airframe side-gust response which would be neglected by considering the sideslip effect alone.

The incremental disturbances of sideslip and yaw-rate produced by the continuous random side gust may be mathematically represented by low-pass-filtered white noise as shown in Fig. 1 where n is a white noise signal of unity power spectrum density and the filter dynamics are[2]

$$T_{v_g}(s) = \sigma_v\left(\frac{L_v}{\pi U_0}\right)^{1/2}\frac{1+[(3)^{\frac{1}{2}}L_v/U_0]s}{(1+L_v/U_0 s)^2} \tag{1}$$

$$R(s) = \frac{s}{1+(3b/\pi U_0)s} \tag{2}$$

In these equations U_0 = equilibrium aircraft airspeed, fps; b = wing span, ft; $L_v = \{145\ h^{1/3}$ below $h = 1750$ ft, 1750 above $h = 1750$ ft$\}$, ft, h = altitude, ft; $\sigma_v^2 = L_v\sigma_w^2/L_w$, (fps)^2 $L_w = \{h$ below 1750 ft, 1750 above 1750 ft$\}$, ft, and σ_w is the vertical gust intensity, fps.

For an airplane with a 38.4 ft span flying 500 ft above ground level at 520 knots Eqs. (1) and (2) become

$$T_{v_g}(s) = \frac{0.6461\sigma_v(1+2.272s)}{(1+1.312s)^2} \tag{3}$$

$$R(s) = \frac{s}{1+.04179s} \doteq s \tag{4}$$

where the side gust intensity is scaled such that $\sigma_v = 1.517\sigma_w$. The incremental gust disturbances of sideslip and yaw rate enter only the aerodynamic terms of the lateral-directional equations of motion of the aircraft.[2] For example, for a typical tactical fighter at the flight condition considered above the equations in stability axes are:

$$\begin{bmatrix} s+1.313 & -.3614 & 10.59 \\ .0119 & s+.5261 & -10.89 \\ -.0003s-.0345 & .998s & s^2+.2257s \end{bmatrix}\cdot\begin{bmatrix} p \\ r \\ \beta \end{bmatrix}$$
$$= \begin{bmatrix} -1.313 & .3614 & -10.59 \\ -.0119 & -.5261 & 10.89 \\ .0003s & -.998s & -.2257s \end{bmatrix}\cdot\begin{bmatrix} p_g \\ r_g \\ \beta_g \end{bmatrix} \tag{5}$$
$$= \begin{bmatrix} -1.313 & (-.3614s+10.59)/877.6 \\ -.0119 & (.5261s-10.89)/877.6 \\ .0003s & (.998s^2+.2257s)/877.6 \end{bmatrix}\cdot\begin{bmatrix} p_g \\ v_g \end{bmatrix}$$

where $v_g = T_{v_g}n$, and the effective roll angular velocity (p_g) due to the spanwise gradient of the continuous random vertical gust has also been included. Again, as with the side gust, the incremental roll-rate disturbance produced by the continuous random vertical gust is mathematically represented by a

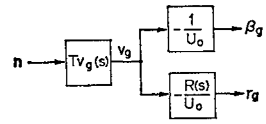

Fig. 1 Mathematical model for the generation of continuous random side gusts and their propagation into equivalent aerodynamic effects.

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low-pass-filtered white noise source; however, it is uncorrelated with that producing the side gust effects. By letting m be another white noise signal of unity power spectrum density, the filter dynamics may be represented[2]

$$T_{p_g}(s) = \frac{\sigma_w[(0.8/L_w U_0)(\pi L_w/4b)^{1/3}]^{1/2}}{1 + (4b/\pi U_0)s} = \frac{1.989(10^{-3})\sigma_w}{1 + .05573s} \tag{6}$$

where $p_g = T_{p_g} m$, and the terms have been evaluated at the flight condition previously introduced.

Using Cramer's Rule, matrix Eq. (5) can be solved to determine the total filtering which processes the m and n white noise signals into continuous random yaw attitude perturbations of the rigid aircraft. This is the lateral tracking error that a pilot could observe during a task such as strafing where the pendulum effect of gunsight depression is not significant. The relative filtering of these side-gust- and vertical-gust-induced lateral tracking errors, scaled to the vertical gust intensity, are shown in Fig. 2 as curves v_g and p_g, respectively. Also shown is the curve β_g which represents the effective side-gust filtering if the directly-induced yaw angular velocity of the side gust is omitted, and only the sideslip disturbance input is considered. Note that the beta-gust alone (β_g) is not a good approximation of the more accurate total side gust effect (v_g), and that the roll-rate disturbance (p_g) induced by a vertical gust can produce larger yaw attitude perturbations at frequencies below one rad/sec than the total side gust (v_g). Since the pilot will often be constrained by stability requirements to a bandwidth of less than one rad/sec when performing lateral tracking by aileron control, the yaw attitude perturbations induced by the roll-rate effect of the vertical gust environment could be the most significant disturbance input.

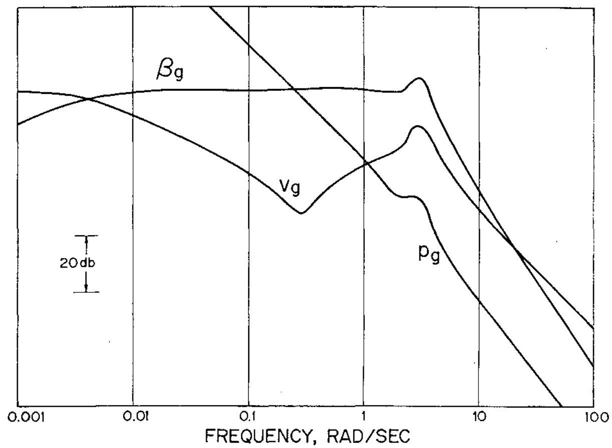

Fig. 2 Relative filtering of white noise to produce rigid-aircraft yaw attitude perturbations originating from: (p_g) roll-rate disturbances due to the spanwise gradient of the vertical gust distribution, (v_g) sideslip and yaw-rate disturbances due to the side velocity at the c.g. and the lengthwise gradient of the side gust distribution, and (β_g) sideslip alone neglecting the accompanying yaw-rate effect.

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